# STA 542 – Homework 6

**Due:** Wednesday, October 28, at the start of class

**Credit.** This homework is graded for a substantive, good-faith attempt. Address every assigned part and show the reasoning behind each answer. Correct answers without supporting work do not constitute a substantive attempt.

**Submission.** Submit one ordered PDF. Mathematical and statistical reasoning must be written legibly by hand; tablet handwriting is welcome. R code, numerical output, tables, and plots may be printed or inserted and appended. Simple code or plots may instead be sketched when exact output is unnecessary. Label every computational exhibit, refer to it in the handwritten response, and explain what it shows.

**Problem 1. Carbon and temperature.** Work through parts (a)–(d) of the Homework 6 workbook, *Carbon and Temperature* (`climate-workbook.html`). Use one temperature record and compare two transformations, as directed there. Include your written answers and the plots, fitted coefficients, and confidence intervals needed to support them.

**Problem 2. Innovation assumptions and Granger causality.** Consider the mean-zero stationary causal VAR(1)

$$X_{t+1} = AX_t + Z_{t+1}, \qquad X_t = \begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix}, \qquad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad \rho(A) < 1.$$

Write $\mathcal{H}_t = (X_s : s \le t)$ for the joint history and $\mathcal{H}_t^{(1)} = (X_{1s} : s \le t)$ for the first coordinate's history. Throughout, Granger causality means Granger causality in mean at horizon one.

(a) Suppose $Z_t$ are iid $N_2(0, \Sigma_Z)$, with $\Sigma_Z$ positive definite. State the condition on $A$ under which coordinate 2 Granger-causes coordinate 1. Does correlation between $Z_{1t}$ and $Z_{2t}$ change this criterion? Explain briefly; you may use Proposition 11.7 of Workbook 11.

(b) Drop the Gaussian assumption from part (a), retaining the stationary VAR(1) above. For the innovations, assume only that, for every $t$, $Z_{t+1}$ has mean zero, finite positive-definite covariance, and is independent of $\mathcal{H}_t$. Find

$$\mathbb{E}[X_{1,t+1} \mid \mathcal{H}_t] \quad \text{and} \quad \mathbb{E}[X_{1,t+1} \mid \mathcal{H}_t^{(1)}],$$

and subtract them. Does $A_{12} = 0$ still imply that coordinate 2 does not Granger-cause coordinate 1? When $A_{12} \neq 0$, give a necessary and sufficient condition on predicting $X_{2t}$ from $\mathcal{H}_t^{(1)}$ for the two forecasts to differ with positive probability. Explain briefly.

**Problem 3. Self-assessment.** Choose one *linear algebra* concept, one *statistics* concept, and one *probability* concept used in Problems 1–2. For each, write two or three sentences explaining where you used it and what you understand about it.